

Scenario of Inflationary Cosmology from the Phenomenological Λ Models

Saibal Ray · P.C. Ray · M. Khlopov · P.P. Ghosh ·
U. Mukhopadhyay · P. Chowdhury

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Abstract Choosing the three phenomenological models of the dynamical cosmological term Λ , viz., $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ where a is the cosmic scale factor, it has been shown by the method of numerical analysis for the considered non-linear differential equations that the three models are equivalent for the flat Universe $k = 0$ and for arbitrary non-linear equation of state. The evolution plots for dynamical cosmological term Λ vs. time t and also the cosmic scale factor a vs. t are drawn here for $k = 0, +1$. A qualitative analy-

S. Ray (✉)

Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700 010,
West Bengal, India
e-mail: saibal@iucaa.ernet.in

P.C. Ray

Department of Mathematics, Government College of Engineering and Leather Technology, Kolkata 700
098, West Bengal, India

M. Khlopov

Center for Cosmoparticle physics “Cosmion”, 125047, Moscow, Russia

M. Khlopov

Moscow Engineering Physics Institute, 115409 Moscow, Russia

M. Khlopov

APC Laboratory, 10, rue Alice Domon et Léonie Duquet 75205 Paris Cedex 13, France

P.P. Ghosh

Tara Brahmamoyee Vidyamandir, Matripalli, North 24 Parganas, Shyamnagar 743 127, West Bengal,
India

U. Mukhopadhyay

Satyabharati Vidyapith, Nabapalli, North 24 Parganas, Kolkata 700 126, West Bengal, India

P. Chowdhury

International Centre for Complex System Studies, West Bengal University of Technology, Kolkata 700
064, West Bengal, India

sis has been made from the plots which supports the idea of inflation and hence expanding Universe.

Keywords General relativity · Cosmological parameter · Inflationary cosmology

1 Introduction

The observations on Supernovae type Ia [1, 2] lead the scientists to the concept of accelerating Universe. The unknown type of energy responsible for this kind of acceleration is known as *dark energy*. Now, there are various types of models related to this dark energy [3, 4] and for those models the expansion rate is also different. The so called *cosmological constant* may be one of them. For the explanation of accelerating Universe, Λ of dynamical character is preferred rather than a constant one. This character can naturally follow from non-linear effects of gravitational and scalar fields, involved in the physical model. In the paper of Ray, Mukhopadhyay and Meng [5], it was shown that among the dynamical models of Λ , the three types $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ are equivalent, where a is the cosmic scale factor of the Robertson-Walker metric and ρ is the matter-energy density. They also analytically established a relationship between the parameters α , β and γ of the respective models. In the present work emphasis has been given to show the equivalence of the same three models of Λ by using the method of numerical analysis of underlying non-linear Einstein equation for the flat Universe ($k = 0$) and for equation of state with arbitrary non-linear dependence on matter-energy density. Another aspect of this work is to make an attempt for visualizing the incidents that occurred during the very early stage of the Universe, especially the feature of *inflation* [6–8].

2 The Field Equations and General Results

The Einstein field equations are given by

$$R^{ij} - \frac{1}{2} R g^{ij} = -8\pi G \left[T^{ij} - \frac{\Lambda}{8\pi G} g^{ij} \right] \quad (1)$$

where $\Lambda = \Lambda(t)$ is the so called *cosmological constant*. Here c , the velocity of light, is assumed to be unity in relativistic units.

Now, let us consider the Friedmann–Lemaître–Robertson–Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (2)$$

where the curvature constant $k = -1, 0, +1$ for open, flat and close models of the Universe respectively.

For the above spherically symmetric metric (2), the non-linear Einstein field equations reduce to the following two equations, respectively the Friedmann equation and the Raychaudhuri equation as

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3}, \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{4}$$

where the fluid pressure p and energy density ρ are related in the form

$$p = \omega\rho^n \tag{5}$$

ω being the equation of state parameter of the polytropic equation of state.

The energy conservation law can be written as

$$8\pi G(p + \rho)\frac{\dot{a}}{a} = -\frac{8\pi G}{3}\dot{\rho} - \frac{\dot{\Lambda}}{3}. \tag{6}$$

Differentiating (3) with respect to time t we get

$$2\left(\frac{\dot{a}}{a}\right)\left[\frac{a\ddot{a} - \dot{a}^2}{a^2}\right] - \frac{2k}{a^3}\dot{a} = \frac{8\pi G}{3}\dot{\rho} + \frac{\dot{\Lambda}}{3}. \tag{7}$$

With the help of (6), (7) reduces to

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \frac{\ddot{a}}{a} = 4\pi G(p + \rho). \tag{8}$$

Now, from (4) with the help of (8) and (5), we get

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{\ddot{a}}{a}\right) + \frac{k}{a^2} = -8\pi G\omega\rho^n + \Lambda. \tag{9}$$

2.1 $\Lambda \sim (\dot{a}/a)^2$

Let us now consider the following dynamical model of Λ which is

$$\Lambda = 3\alpha\left(\frac{\dot{a}}{a}\right)^2. \tag{10}$$

Employing (10) in (3) one immediately obtains

$$\rho = \frac{3}{8\pi G}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] - \frac{3\alpha}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2. \tag{11}$$

Now taking $n = 1$ and using (11), (9) reduces to the form

$$\ddot{a} = -(3\omega + 1)\left[\frac{\dot{a}^2}{2a} + \frac{k}{a}\right] + 3\alpha(1 + \omega)\frac{\dot{a}^2}{2a}. \tag{12}$$

2.2 $\Lambda \sim \ddot{a}/a$

As a second dynamical model we now start with

$$\Lambda = \beta\left(\frac{\ddot{a}}{a}\right). \tag{13}$$

By the use of (13) in (3), we get the value for ρ as

$$\rho = \frac{3}{8\pi G} \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] - \frac{\beta}{8\pi G} \left(\frac{\ddot{a}}{a} \right). \quad (14)$$

For $n = 1$, by the use of (14), (9) finally gives the value of \ddot{a} as

$$\ddot{a} = \frac{(1 + 3\omega)}{[(\omega + 1)\beta - 2]} \left[\frac{\dot{a}^2}{a} + \frac{k}{a} \right]. \quad (15)$$

2.3 $\Lambda \sim \rho$

Let us now take the third form of phenomenological Λ as

$$\Lambda = 8\pi G\gamma\rho \quad (16)$$

where the constraint is such that $\gamma > 0$. Using the value of Λ given in (16), we obtain from (3) the value of ρ as

$$\rho = \frac{3}{8\pi G(1 + \gamma)} \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right]. \quad (17)$$

Using (16) and (17), we get from (9) the value of \ddot{a} for $n = 1$ as

$$\ddot{a} = \frac{2\gamma - 3\omega - 1}{2(\gamma + 1)} \left[\frac{\dot{a}^2}{a} + \frac{k}{a} \right]. \quad (18)$$

In this connection it is to be noted here that Ray, Mukhopadhyay and Meng [5] have shown that the three forms $\Lambda = 3\alpha(\dot{a}/a)^2$, $\Lambda = \beta(\ddot{a}/a)$ and $\Lambda = 8\pi G\gamma\rho$, as expressed in (10), (13) and (16), are equivalent for $k = 0$. They found through analytical method that the parameters involved in the three dynamical relations are connected by

$$\alpha = \frac{\beta(1 + 3w)}{3(\beta w + \beta - 2)} = \frac{\gamma}{1 + \gamma}. \quad (19)$$

This means that it is possible to find out the identical physical features of others if any of those three phenomenological Λ relations is known.

In this connection it is, however, interesting to note here that for linear dependence on energy density in equation of state (5), corresponding to $n = 1$, (15) of Case 2.2 and (18) of Case 2.3 are equivalent for $k = +1, 0, -1$ when the relation

$$\frac{(1 + 3\omega)}{[(\omega + 1)\beta - 2]} = \frac{2\gamma - 3\omega - 1}{2(\gamma + 1)} \quad (20)$$

holds good.

3 Graphical Presentation of the Results

Now using the method of numerical analysis, let us try to study the variation of the cosmological parameter Λ and the scale factor a with time which are shown in the following plots.

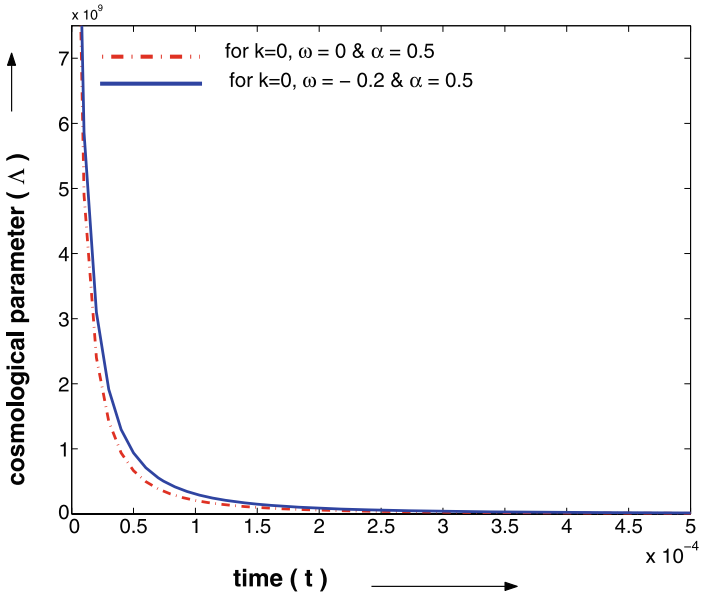


Fig. 1 Case 2.1 related to model $\Lambda = 3\alpha(\dot{a}/a)^2$ shows variation of cosmological parameter for $k = 0$

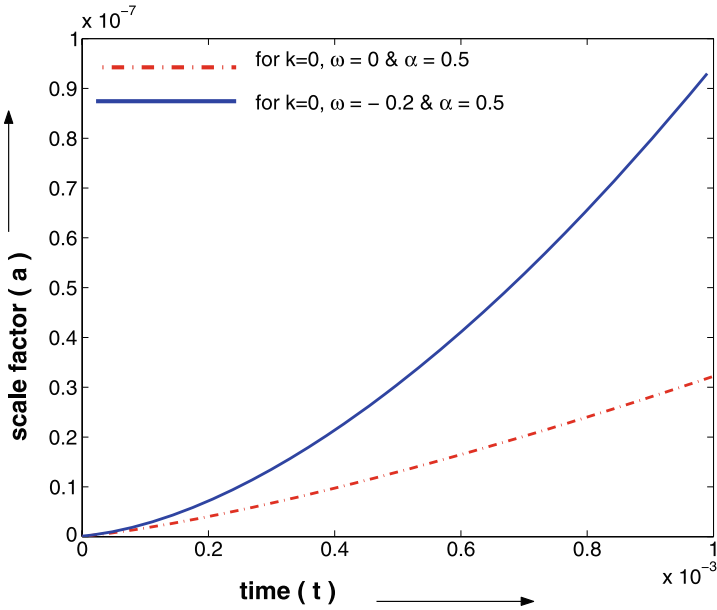


Fig. 2 Case 2.1 related to model $\Lambda = 3\alpha(\dot{a}/a)^2$ shows variation of scale factor for $k = 0$

From a close observation of the graphical plots, viz., Figs. 1, 5 and 9, one can find out equivalence of the three models with respect to time variation of Λ while Figs. 2, 6 and 10 exhibit equivalence of the same three models with respect to time variation of a for $k = 0$

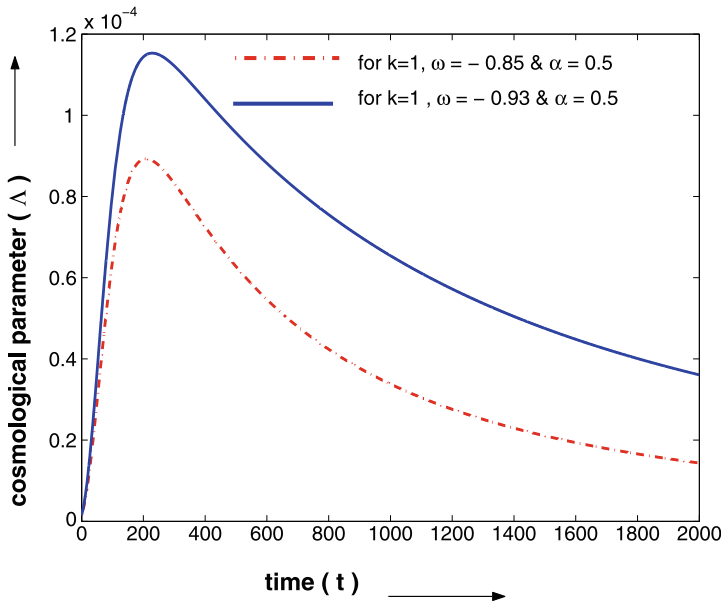


Fig. 3 Case 2.1 related to model $\Lambda = 3\alpha(\dot{a}/a)^2$ shows variation of cosmological parameter for $k = +1$

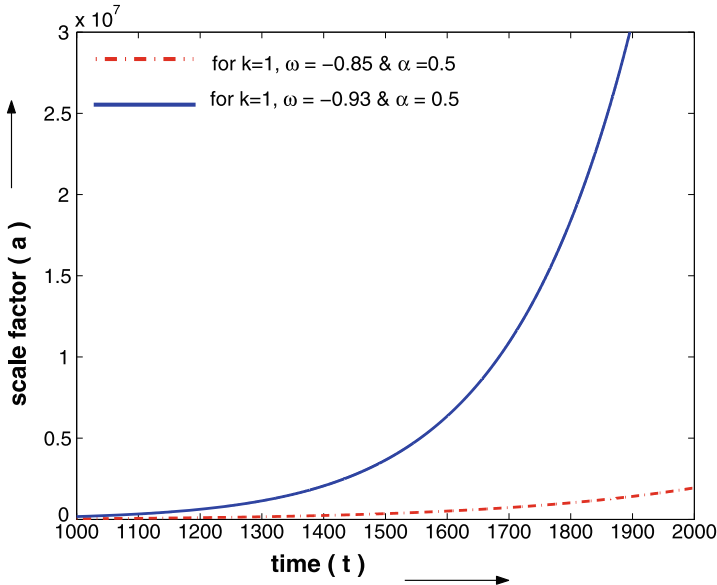


Fig. 4 Case 2.1 related to model $\Lambda = 3\alpha(\dot{a}/a)^2$ shows variation of scale factor for $k = +1$

as obtained in analytical method by Ray, Mukhopadhyay and Meng [5]. It can also be observed that Figs. 7 and 11 show equivalence of $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models with respect to variation of Λ with time for $k = +1$. However, the behavior of Λ with respect to time for $\Lambda \sim (\dot{a}/a)^2$ model is quite different. For $\Lambda \sim (\dot{a}/a)^2$ model we observe an abrupt increase

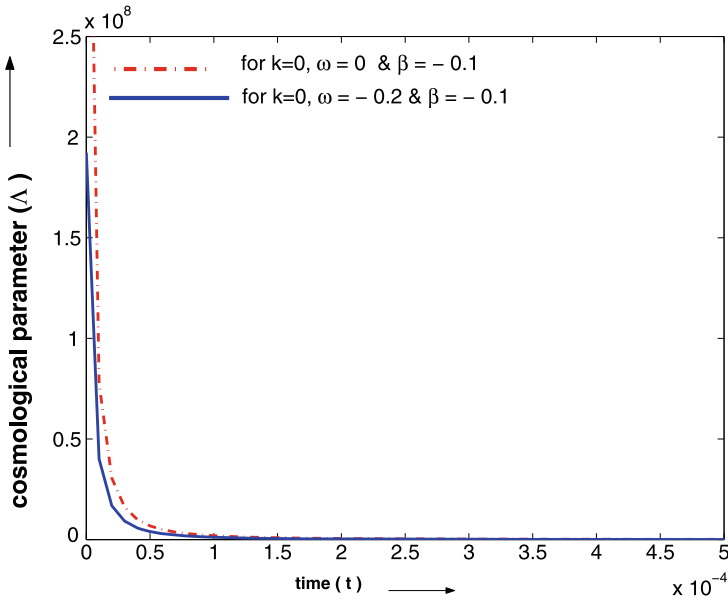


Fig. 5 Case 2.2 related to model $\Lambda = \beta(\ddot{a}/a)$ shows variation of cosmological parameter for $k = 0$

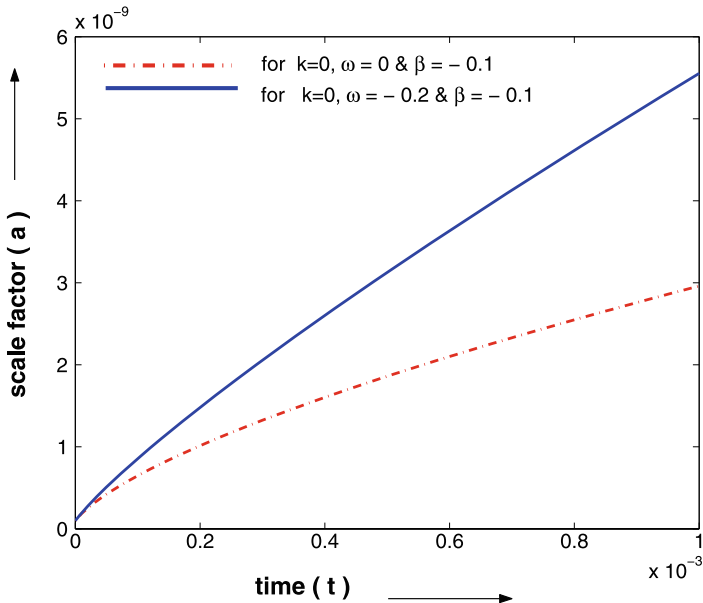


Fig. 6 Case 2.2 related to model $\Lambda = \beta(\ddot{a}/a)$ shows variation of scale factor for $k = 0$

of the cosmological parameter within a very short period of time and then a comparatively slower decrease of it. This abrupt rise of Λ may be interpreted as the driving force behind inflation because time variation of the scale factor a for the same values of $k, \omega, \alpha,$

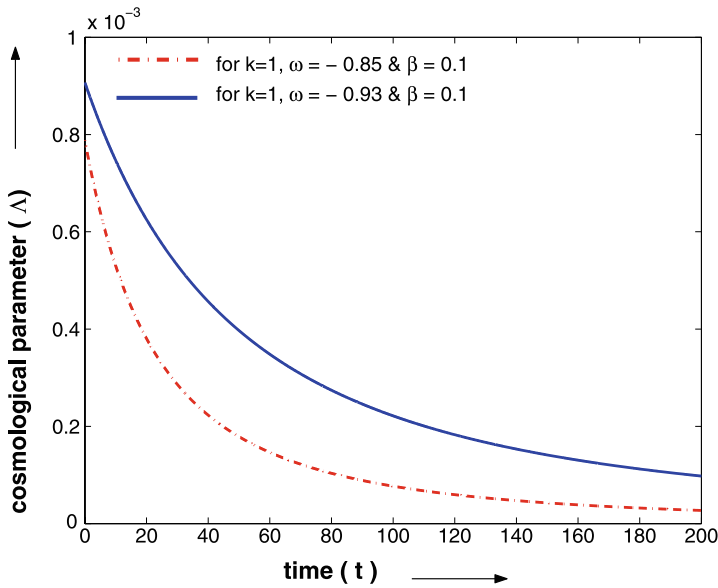


Fig. 7 Case 2.2 related to model $\Lambda = \beta(\ddot{a}/a)$ shows variation of cosmological parameter for $k = +1$

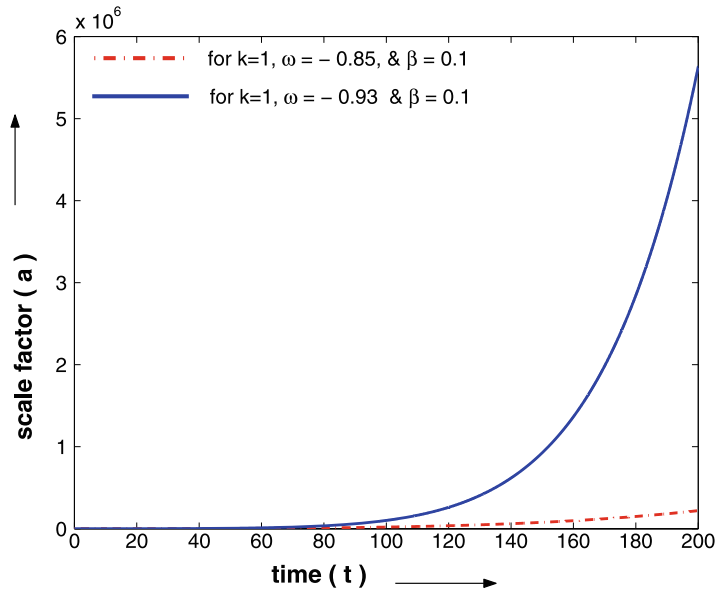


Fig. 8 Case 2.2 related to model $\Lambda = \beta(\ddot{a}/a)$ shows variation of scale factor for $k = +1$

β and γ show a very sharp increase as depicted in Figs. 4, 8 and 12. Moreover, the sudden jump in the value of Λ exhibited in Fig. 3 for $k = 1$ shows a clear indication of the role of the dark energy candidate Λ as repulsive pressure in connection to inflationary phase of the Universe [6–8] and also may be interpreted as a numerical manifestation of the idea

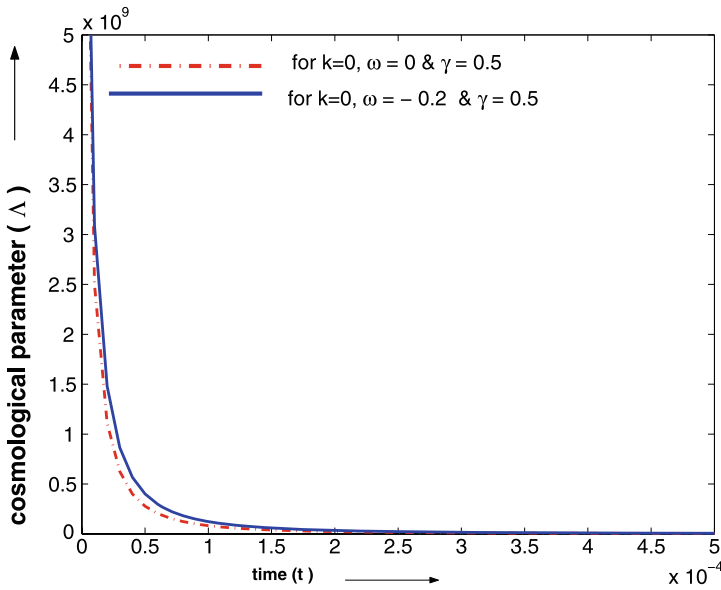


Fig. 9 Case 2.3 related to model $\Lambda = 8\pi G\gamma\rho$ shows variation of cosmological parameter for $k = 0$

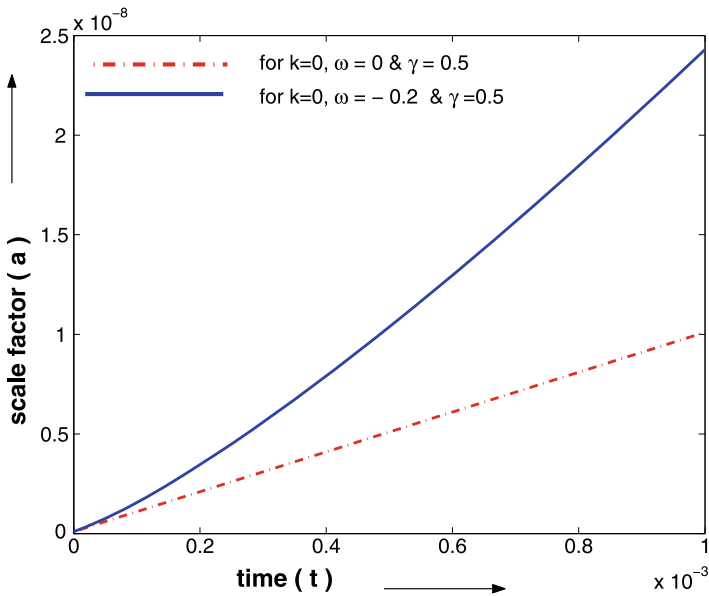


Fig. 10 Case 2.3 related to model $\Lambda = 8\pi G\gamma\rho$ shows variation of scale factor for $k = 0$

that dark energy is responsible for making the Universe flat during inflation. This is because immediately after attaining a peak value, Λ has dropped down. So, it is quite natural to think of that a huge amount of dark energy was used up for triggering the exponential cosmic expansion as well as for removing the curvature of the Universe. Interestingly, it

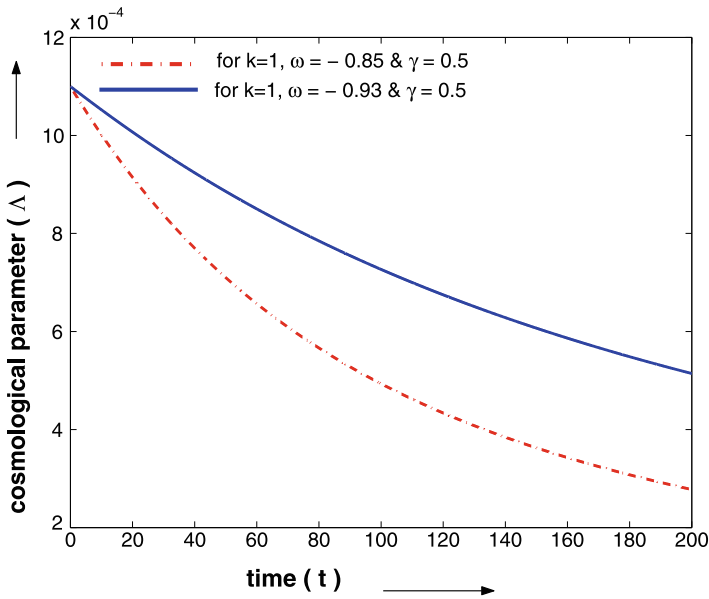


Fig. 11 Case 2.3 related to model $\Lambda = 8\pi G\gamma\rho$ shows variation of cosmological parameter for $k = +1$

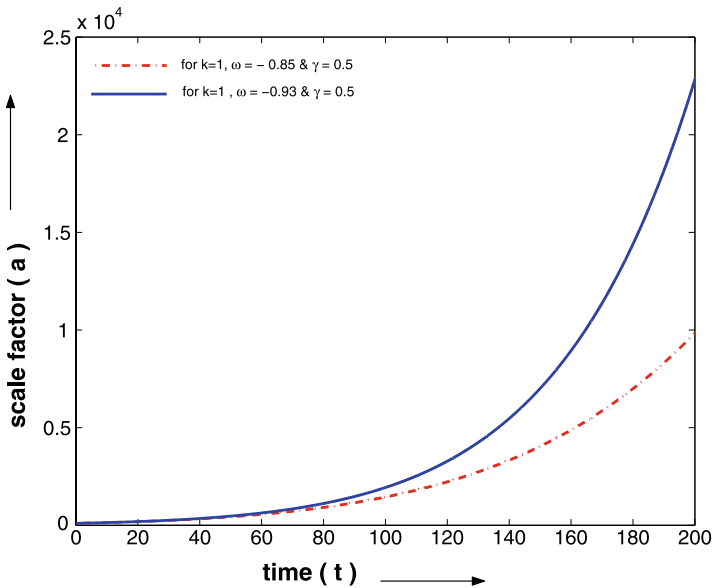


Fig. 12 Case 2.3 related to model $\Lambda = 8\pi G\gamma\rho$ shows variation of scale factor for $k = +1$

is to be noted here that this type of qualitative variation of Λ with time during inflation was unavailable in the analytical method by Ray, Mukhopadhyay and Meng [5]. However, due to lack of advanced computing facility, in the present investigation we could not quan-

tify the time duration of inflation which has been suggested in the literature as 10^{-35} to 10^{-32} sec [9].

The present work shows that although phenomenological models do not originate from any quantum field theory, yet at least in some cases they can successfully reflect the present cosmological picture. This is another positive side of this investigation. In particular taking the values of $\Omega_{m0} = 0.330 \pm 0.035$ [1–4], the ranges of the values of model parameters $\alpha_0, \beta_0, \gamma_0$, corresponding to the observed accelerated expansion, are obtained as $0.635 \geq \alpha_0 \geq 0.705$, $3.417 \geq \beta_0 \geq 4.674$ and $1.739 \geq \gamma_0 \geq 2.389$.

Using the above-mentioned values of Ω_{m0} , the ranges of the present values of the cosmological parameter Λ_0 are obtained as $1 \times 10^{-35} \text{ s}^{-2} - 2 \times 10^{-35} \text{ s}^{-2}$, which agree with the results of Carmeli [10] and Carmeli and Kuzmenko [11], where they obtain the value $1.934 \times 10^{-35} \text{ s}^{-2}$.

4 Conclusions

In the present investigation, instead of finding out exact solution of the ordinary non-linear differential equation, the method of numerical analysis has been adopted for the three phenomenological models of Λ , viz., $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$. However, the main idea of the article i.e. the study of numerical solutions for cosmological problems in absence of particular or general analytic solutions is not a new idea. Interesting literature in this aspect are available, where numerical cosmology with the Cactus code [12] or testing the Cactus code on exact solutions of the Einstein field equations [13] have been performed.

The dynamical nature of Λ and its time variation of the present investigation can find physical origin in non-linear effects of gravity (as it was the case for nonlinear R^2 term induced by vacuum polarization in one of the first inflationary models [14]) or of scalar fields, involved in the model (as it took place in the of self consistent inflation [15–17]). The interesting features of this investigation can be put in the following way:

- (1) The time variation of Λ and a for $k = 0$ support the work of Ray, Mukhopadhyay and Meng [5] so far as the equivalence of the three chosen phenomenological models are concerned.
- (2) It has been possible to establish that $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models are also equivalent for $k = 1$.
- (3) Finally, a bonus obtained from the present numerical work is the qualitative visualization of inflationary scenario of the Universe through the variation of Λ .

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